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Frequency Up-Shift for Cyclotron Wave Instability on a Relativistic Electron Beam

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The axial drift of a relativistic electron beam along a uniform field is shown theoretically to lead to significant frequency up-shifts for linear cyclotron-wave instabilities driven by velocity-space anisotropy. This mechanism suggests novel means for development of devices for generation of intense far infra-red power.

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FREQUENCY UP-SHIFT FOR CYCLOTRON WAVE INSTABILITY ON A RELATIVISTIC ELECTRON BEAM

Mechanisms for obtaining gain from free-electron systems are being increasingly exploited in the millimeter and infrar-red regions of the electromagnetic spectrum. Reviews have recently appeared of developments [1], theory [2], and recent Soviet results [3] with electron cyclotron masers. Stimulated scattering processes have also been reviewed [4] as novel sources or intense short-wavelength radiation, with an advantage over the cyclotron maser devices of the absence of an intense resonance magnetic field. Relativistic beams undulating in periodic magnetic wigglers have provided measurable gain [5] and substantial power [6] at wavelengths in the micron range.

The theoretical result presented in this Letter suggests that a substantial Doppler up-shift may allow extension of cyclotron resonance devices to shorter wavelengths. In physical terms, a fixed observer viewing an approaching electron beam in which an oscillation is present at frequency Ω_c will see the radiation in his own frame at a Doppler-shifted frequency $\omega = \Omega_c + kv_z$, where v_z is the stream speed and k is the oscillation wavenumber. For a slow wave, such as the right-hand circularily polarized cyclotron wave, kv_z can be much larger than Ω_c, so that $\omega \gg \Omega_c$. Velocity-space anisotropy is known to drive an instability for this mode, but to our knowledge no analysis of the appropriate linear relativistic dispersion relation for a drifting anisotropic electron distribution has appeared [7]. Our results show that large wave growth can be obtained with an arbitrarily large frequency up-shift for a cold beam, and that substantial growth with frequency up-shifts of over a factor of ten are achievable with finite temperature beams.

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The appropriate dispersion relation has been derived previously[8]. For the right-hand circularily polarized plane electromagnetic wave [$\sim \exp(-i\omega t + ikz)$] propagating along a uniform static magnetic field ($B\,\mathbf{e}_z$), we have

$$\omega^2 - k^2c^2 = 2\pi\omega_p^2 \int_0^\infty p_\perp\, dp_\perp \int_{-\infty}^\infty dp_z\, \gamma^{-1} f_o(p_\perp, p_z)$$

$$\left[\frac{\omega - kp_z/\gamma m}{\omega - kp_z/\gamma m - \Omega/\gamma} - \frac{p_\perp^2(\omega^2 - k^2c^2)}{2\gamma^2 m^2c^2(\omega - kp_z/\gamma m - \Omega/\gamma)^2}\right]. \quad (1)$$

where m is the rest electron mass, $\omega_p = 4\pi\ n\,e^2/m$, is the plasma frequency, $p_\perp$, p_z are the moments perpendicular and parallel to the magnetic field, respectively, $\gamma = (1 + p_\perp^2/m^2c^2 + p_z^2/m^2c^2)^{1/2}$, $\Omega = eB/mc$, is the nonrelativistic cyclotron frequency, and $f_o(p_\perp, p_z)$ is normalized such that $\int dp_\perp\, p_\perp \int dp_z\, f_o(p_\perp, p_z) = 1$. The need to retain relativistic effects in deriving and analyzing (1) has been discussed in Ref. 8. This dispersion relation predicts instability driven by the cyclotron maser mechanism (azimuthal bunching) for fast waves, and by the Weibel mechanism (axial bunching) for slow waves. In the present work we focus on slow waves so as to take greatest advantage of the aforementioned Doppler up-shift.

We first obtain an analytical result from (1) for a cold beam, e.g. $f_o(p_\perp, p_z) = (2\pi p_\perp)^{-1}$ $\delta(p_\perp - p_{\perp o})\,\delta(p_z - p_{zo})$. Then (1) reduces to

$$\omega^2 - k^2c^2 = \frac{\omega_p^2}{\gamma_o}\left[\frac{\omega - kv_{zo}}{\omega - kv_{zo} - \Omega_c} - \frac{\beta_{\perp o}^2(\omega^2 - k^2c^2)}{2(\omega - kv_z - \Omega_c)^2}\right]. \quad (2)$$

where $\gamma_o = (1 + p_{\perp o}^2/m^2c^2 + p_{zo}^2/m^2c^2)^{1/2}$, $v_{zo} = p_{zo}/\gamma_o m$, $\beta_{\perp o} = p_{\perp o}/\gamma_o mc$, and $\Omega_c = \Omega/\gamma_o$. An approximation for the unstable root $\omega(k)$ can be easily found from (2) in the short wavelength limit, i.e. as $k \to \infty$. If we assume that $\omega - kv_{zo}$ remains finite as $k \to \infty$, then in the limit $k \to \infty$ Eq. (2) reduces to

$$(\omega - kv_{z0} - \Omega_c)^2 + \omega_p^2 \beta_{\perp 0}^2/2\gamma_0 = 0$$

which gives

$$\omega = kv_{z0} + \Omega_c + i\,\beta_{\perp 0}\,\omega_p/(2\,\gamma_0)^{1/2}. \tag{3}$$

Eq. (3) justifies aposteriori our assumption that $\omega - kv_{z0}$ remain finite as $k \rightarrow \infty$. Thus ω given by (3) is a valid solution of (2) in the short wavelength limit.

According to (3), wave growth at a rate $\omega_p\, \beta_{\perp 0}/(2\,\gamma_0)^{1/2}$ can take place at a frequency which is arbitrarily high, as compared with the cyclotron frequency. This growth rate is independant of both frequency and magnetic field. As an example, for $\beta_{\perp 0} = 0.1$ and $\gamma_0 = 2$, we have a growth rate (ω_i) of $0.05\omega_p$ for a wave frequency ten times higher than the relativistic cyclotron frequency Ω_c, with a wavenumber of $9\Omega_c/v_{z0}$.

We now consider the more realistic case of a finite energy spread on the beam. As shall be seen, the main effect of this energy spread is to reduce the wave growth rate at the higher frequencies. To study this, we have obtained numerical solutions for $\omega(k)$ from (1) for a distribution function $f_0(p_z, p_\perp) = K \exp[-(p_z - p_{z0})^2/(\Delta p_z)^2 - (p_\perp - p_{\perp 0})^2/(\Delta p_\perp)^2]$ where K is a normalization constant. Examples are shown in Fig. 1 for $\gamma_0 = 2$, $p_{\perp 0} = p_{z0}$, $\Delta p_\perp = \Delta p_z$, and for three values of ω_p/Ω_c, namely 4, 1, and 0.2. In the figure, the parameter $T = \Delta p_\perp/p_{\perp 0} = \Delta p_z/p_{z0}$ measures the relative thermal spread. The case $T = 0$ corresponds to that analyzed above for the cold beam. Indeed, results in the limit of large k in each case show excellent agreement with the approximate analytic form (3). We can pick several examples from the figure to bring out the magnitude of the growth and of the frequency up-shift. From Fig. 1a, where $\omega_p/\Omega_c = 4$, we find for $T = 0.1$ a growth rate of $0.64\ \Omega_c$ at a frequency of $16\ \Omega_c$, with a wavenumber of $24.8\ \Omega_c/c$. From Fig. 1b, where $\omega_p/\Omega_c = 1$, we find for $T = 0.05$, a growth rate of $0.12\ \Omega_c$ at a frequency of $10.2\ \Omega_c$ with a wavenumber of $15\ \Omega_c/c$. From Fig. 1c, where $\omega_p/\Omega_c = 0.2$, we find for $T = 0.02$, a growth rate of $0.034\ \Omega_c$ at a frequency of $4.8\ \Omega_c$

with a wavenumber of 6.2 Ω_c/c. These examples all show substantial growth at frequencies shifted well above the relativistic cyclotron frequency Ω_c. Increasing T or decreasing ω_p/Ω_c mitigates against growth, so that the up-shift phenomenon discussed here is best exploited with cold high-density beams. Thus, from the example cited above taken from Fig. 1a, one could imagine designing two oscillators: one at millimeter wavelength, and one at far infra-red wavelength. With a 500 kV electron beam launched with $p_{zo} = p_{\perp o}$ in a 5 kG magnetic field, the analysis predicts a growth rate of 28 (nsec) $^{-1}$ at a wavelength of 2.7 mm using a beam of current density 40 kA/cm^2. In a 50 kG magnetic field, the analysis predicts a growth rate of 280 (nsec) $^{-1}$ at a wavelength of 270 microns using a beam of current density 4 MA/cm^2. The thermal spread of the beam is 10% for both examples. While the second of these examples may be beyond the capabilities of present technology, interest in exploiting the Doppler up-shift principle discussed in this Letter for generation of intense far infra-red power may serve to encourage further development of the necessary intense focussed electron beams.

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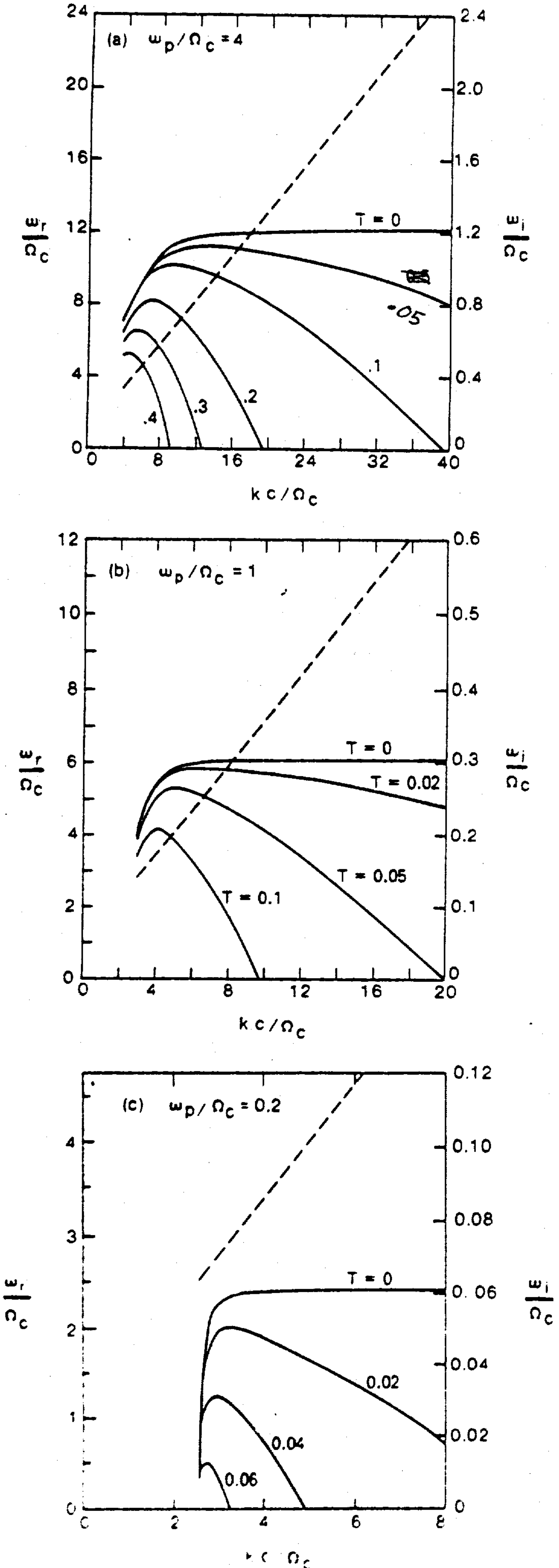

Fig. 1 – Frequency ω_r (dashed curves, normalized to Ω_c) and growth rate ω_i (solid curves, normalized to Ω_c versus kc/Ω_c for various beam temperatures $T(= \Delta p_\perp/p_{\perp 0} = \Delta p_z/p_{z0})$ and three values of ω_p/Ω_c: (a) $\omega_p/\Omega_c = 4$, (b) $\omega_p/\Omega_c = 1$, and (c) $\omega_p/\Omega_c = 0.2$. In each figure, the frequency varies only slightly with the temperature, and is well represented by a single curve.